$$u_{sl} = \frac{3}{2}\mu \frac{1-(1-q)\alpha_0^-}{\alpha_1^+ - (1-q)\alpha_1^-}\left[1 - \frac{2+q+(3q-2)\cdot\alpha_0^-}{1-(1-q)\alpha_0^-} b\rho/10\right]\frac{d}{dy}\ln T, \qquad \mu = \eta/\rho. \tag{10}$$

In the limit cases of pure diffusion (q = 1) and pure specular (q = 0) reflection, we obtain

$$u_{sl} = 1.69\mu(1 - 0.337b\rho)\frac{d}{dy}\ln T, \tag{11}$$

$$u_{sl} = \frac{3}{4}\mu(1 - 0.2b\rho)\frac{d}{dy}\ln T. \tag{12}$$

Formulas (10)-(11) differ from the corresponding formulas for a rarefied gas by terms proportional to $b\rho$. As $\rho \to 0$, expressions (10)-(12) go over into the corresponding expressions in [2].

Let us determine the thermal slip coefficient as follows:

$$u_{sl} = k_{sl}\mu\frac{d}{dy}\ln T.$$

Then it follows from (10)-(12) that the thermal slip coefficient k_{sl} is less in dense than in rarefied gases.

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PECULIARITIES IN THE ONE-DIMENSIONAL MODEL OF RADIANT HEAT EXCHANGE

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UDC 536.3.001.24

Radiant heat exchange is considered in a one-dimensional model. The role of internal heat transfer is considered. Maximum and minimum heat liberation values are determined. A method for calculation is proposed.

The most widely used model for study of radiant heat exchange in a furnace is the one-dimensional model. In such a model the furnace operating space is likened to a channel, along which the exhaust gases move. The gas temperature along the directions perpendicular to the motion is assumed constant. There is no theoretical justification for the use of such a model.

We will write the energy equation of an elementary volume in the following form:

$$-\frac{W}{f}\frac{dT}{dz} + q_{\mathrm{c}} = \frac{h^{\mathbf{r}}_{V-F}(z)}{f}\varepsilon_{\mathrm{s}}\sigma_0(T^4 - T^4_{\mathrm{s}}) - \frac{1}{f}\int_0^l q_{\mathrm{it}}(z, z_k)\,dz_k, \tag{1}$$

where

$$h^{\mathbf{r}}_{V-F}(z) = \frac{H^{\mathbf{r}}_{V-F}(z, H)}{\Delta z}, \quad \Delta z \to 0; \tag{2}$$

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$H^r_{V-F}(z, H)$ is the resolving coefficient for mutual radiant heat exchange between the elemental volume and the surface H;

$$q_{it}(z, z_k) = h^r_{V-V}(z, z_k)(T^4(z) - T_s^4(z_k)); \tag{3}$$

$$h^r_{V-V} = \lim_{\Delta z, \Delta z_k \to 0} \frac{H^r_{V-V}(z, z_k)}{\Delta z, \Delta z_k},$$

where $H^r_{V-V}(z, z_k)$ is the resolving coefficient for mutual radiant heat exchange between elementary volumes. From Eq. (3) it follows that

$$q_{it}(z, z_k) = -q_{it}(z_k, z).$$

The quantity $Q_{it} = q_{it}(z, z_k)\Delta z \Delta z_k$ defines the quantity of heat obtained by the elemental volume Δz_k from another volume. We will term this quantity the internal transfer heat (ITH).

If we have two modes of heat exchange with identical values of q_{it} for all the gases z, z_k, we then assume that the ITH is identical for all. But if for all or some pairs of z, z_k values for the two modes the quantity q_{it} of one is larger than that of the other, while for all remaining pairs the quantities are equal, we assume that in such a mode the ITH is higher. We introduce the notation

$$\int_a^b dz \int_c^d q_{it}(z, z_k)\, dz_k = I(a,\ b;\ c,\ d), \quad I_z = I(0,\ z;\ 0,\ l).$$

We now use the identity $I(0, z; 0, z) \equiv 0$. From the definition of I we find

$$I_z = I(0,\ z;\ 0,\ z) + I(0,\ z;\ z,\ l). \tag{4}$$

The first term of the right-hand side is equal to zero, the second is the quantity of heat obtained by the chamber segment from $0-z$ from the segment $z-l$. For a temperature curve falling along the channel this will be a negative quantity, so that I_z will also be negative, while for temperature rising along the channel, these values will be positive. Thus, with increase in ITH, the absolute value of I_z increases.

By the definition of the above integrals

$$I_l = I_z + I(z,\ l;\ 0,\ l). \tag{5}$$

Since $I_l = 0$, the values of the integrals I_z and $I(z, l; 0, l)$ are equal in magnitude and opposite in sign, i.e., for a falling temperature curve $I(z, l; 0, l)$ is positive, and for an increasing curve, negative.

We will integrate Eq. (1) over the entire chamber length and over the length from zero to z. Considering that $I_l = 0$, we obtain

$$W(T_1 - T_{ex}) + Q_c = Q_t, \tag{6}$$

$$W[T_1 - T(z)] + Q_c(z) = Q_t(z) - I_z, \tag{7}$$

where

$$Q_t(z) = \int_0^z h^r_{V-F}(z)\, \varepsilon_s \sigma_0 (T^4 - T_s^4)\, dz \tag{8}$$

is the heat given off by the gases in the segment $0-z$ to the heating surface.

Equation (6) is the balance equation for the entire chamber, while Eq. (7) is the balance equation for the gas volume in the interval $0-z$..

Internal transfer heat may be produced by other methods: radiation of energy within the operating space, direct transfer of a quantity of heat from one place in the working space to another, transfer of a mass of gas at different temperature, thermal conductivity.

We will consider how the ITH affects heat exchange. We will consider two heat-exchange modes with different ITH and an identical chemical heat liberation potential. The temperature curves of both modes must intersect each other, since if the curve for mode 1 lies above that of mode 2, then according to Eq. (8), $Q_{t1} > Q_{t2}$, while under the conditions postulated, $T_{ex1} > T_{ex2}$, which is not compatible with Eq. (6). We will now consider modes with intersecting temperature curves which fall along the chamber length. We integrate Eq. (1) over the segment z−1, where z is the coordinate of the last intersection of the temperature curves:

$$I(z,\ l;\ 0,\ l) = Q_t(z-l) - W[T(z) - T_{ex}] - Q_c(z-l). \tag{9}$$

From Eq. (5) $I(z, l; 0, l)$ is positive, and for heat exchangers with a high ITH, it will be larger; consequently the difference $Q_t(z-l) - W[T(z) - T_{ex}]$ will be larger, which is possible only in the case where the curve over segment $z - l$ for the heat exchanger with higher ITH goes above the curve for the exchanger with lower ITH, from whence it follows that the temperature of the exiting gases will be higher, and the heat liberation smaller. Thus, for the case of a decreasing temperature dependence, an increase in ITH decreases heat liberation. Therefore, maximum heat liberation will occur in the absence of ITH ($q_{it} = 0$). On the other hand, at maximum ITH, which corresponds to identical temperature over the chamber, heat liberation will be at a minimum. Analogous considerations will show that for increasing temperature curves ITH increases heat liberation, so that the most unfavorable condition from the standpoint of heat liberation is the case without ITH, and the most favorable condition will be maximum ITH, i.e., the picture is reversed from that for heat exchangers with decreasing temperature curves.

If we neglect ITH, then the heat-exchange equation will have the form

$$\frac{dT}{dz} - \frac{fq_c}{W} + \frac{h^r_{V-F}(z)}{W}\varepsilon_s\sigma_0(T^4 - T_s^4) = 0. \tag{10}$$

The quantity $h^r_{V-F}(z)$ is a function of z. According to the principle of additiveness,

$$\int_0^l h^r_{V-F}(z)\,dz = H^r_{V-F}(H, V).$$

The product of this quantity with $\varepsilon_s\sigma_0$, according to the physical meaning of the coefficients, will equal $\sigma_a H$. Making use of these facts, we then find

$$\int_0^l h^r_{V-F}(z)\,dz = \frac{\sigma_a H}{\varepsilon_s\sigma_0} = \frac{\varepsilon_a}{\varepsilon_s} H. \tag{11}$$

In the particular case $q_c = 0$

$$\int_{T_2}^{T_1} \frac{dT}{T^4 - T_s^4} = \frac{\varepsilon_s\sigma_0}{W}\int_0^l h^r_{V-F}(z)\,dz = \frac{\sigma_a H}{W}. \tag{12}$$

For $T_s = 0$

$$\theta_{ex} = 1/\sqrt[3]{1 + 3\pi_r}, \tag{13}$$

where $\pi_r = \varepsilon_a\sigma_0 T_1^3/W$ is the radiation parameter. This case is of interest because a solution can be obtained without any assumptions as to the form of the z dependence of $h^r_{V-F}(z)$. If q_c is not equal to zero, then for an exact solution it is necessary to know this dependence, although $h^r_{V-F}(z)$ usually varies little with length so we may use some mean value $h^r_{V-F}(z)_m = (\varepsilon_a/\varepsilon_s)\psi\Pi$. Then

$$\frac{dT}{dz} - \frac{fq_c}{W} + \frac{\sigma_a\psi\Pi}{W}(T^4 - T_s^4) = 0. \tag{14}$$

For maximum ITH the heat-exchange equation has the form

$$H\sigma_a(T_{ex}^4 - T_s^4) = Q_c + W(T_1 - T_{ex}), \tag{15}$$

and for $Q_c = 0$

$$\pi_r(\theta_{ex}^4 - \theta_s^4) + \theta_{ex} - 1 = 0. \tag{16}$$

From the above it follows that for heat exchangers with decreasing temperature curves the maximum heat liberation is determined by Eqs. (10) and (14), and minimal liberation by Eq. (15) or Eq. (16), while for increasing temperature curves maximum heat liberation is determined by Eq. (15), and the minimum value by Eq. (10).

Figure 1 shows the temperature of the exiting gases as a function of π_r for a decreasing temperature curve for the cases of maximum (θ_1) and minimum (θ_2) heat liberation. Moreover, the same dependence is shown for the case of heat exchange in a plane channel with consideration of internal heat transfer by radiation, as obtained to a high accuracy by zone method calculations. At the channel input and output the gases are assumed bounded by adiabatic surfaces. Curves 4 and 5 give the maximum loss in heat liberation due to mixing, first as a fraction of WT_1, $y_1 = \theta_2 - \theta_1$, and second, as a fraction of the heat used in the chamber $y_2 = (\theta_2 - \theta_1)/(1 - \theta_1)$. A comparison of curves 1 and 3 shows that the loss in heat liberation due to ITH by radiation is small.

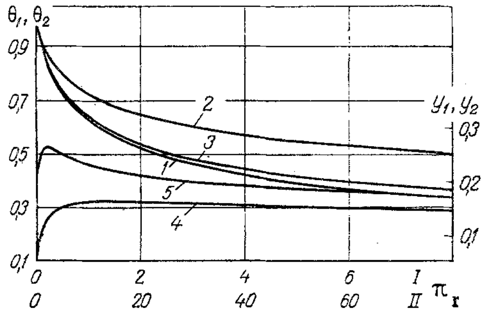

Fig. 1. Exiting gas temperature: 1) without ITH (scale I); 2) with maximum ITH (scale I); 3) with ITH by radiation; 4) y_1 (scale II); 5) y_2 (scale II).

At low π_r it is equal to zero. At some value of π_r it begins to increase with increase in π_r, reaching a maximum value of about 3.5%.

The energy equation can also be written in the following form [1, 2]:

$$-\frac{W}{f}\frac{dT}{dz}+q_c-\frac{dI_r(z)}{dz}=\frac{E_r(z)\psi\Pi}{f}, \tag{17}$$

where $I_r(z)$, $E_r(z)$ are the resultant radiant fluxes along the channel wall and along the heating surface, Eq. (1).

We integrate Eq. (17) with limits 0 and z:

$$W(T_1-T(z))+Q_c(z)+(I_1-I(z))f=\psi\Pi\int_0^z E_r(z)\,dz. \tag{18}$$

Using Eqs. (1), (3), (17), we obtain

$$E_r\psi\Pi=h^r_{V-F}(z)\,\varepsilon_s\sigma_0(T^4-T_s^4)-\int_0^l h^r_{V-V}(T_k^4-T^4)\,dz_k-f\,\frac{dI_r}{dz}. \tag{19}$$

Considering that the initial section is adiabatic ($I_{r1}=0$), we integrate Eq. (19) with limits 0 and z:

$$Q_{hs}(z)=\psi\Pi\int_0^z E_r\,dz=Q_t(z)-I_z-fI_r(z). \tag{20}$$

In Eq. (20) the term Q_t is the value of the heat liberation by the gas in the segment 0–z, while $Q_{hs}(z)$ is the resultant heat exchange of the heating surface over the same segment.

For quantitative evaluations, computer calculations of heat exchange were performed by the zone method for a gas flow moving in a plane channel 1 m in height and 4 m in length, bounded above and below by black heating surfaces with end faces formed by adiabatic walls perpendicular to the flow direction and permeable to the gas. It was assumed that the absorption coefficient of the medium $\alpha = 0.2\ \mathrm{m}^{-1}$, $W/\sigma_0T_1^3 = 2$. Calculations were performed for a 1-m channel width.

Figure 2 shows the quantities $(T_1-T(z))\frac{W}{\sigma_0T_1^4}$ (curve 1), $\frac{Q_t(z)}{\sigma_0T_1^4}$ (2), $\frac{\psi\Pi}{\sigma_0T_1^4}\int_0^z E_r dz$ (3), $\frac{I_r}{\sigma_0T_1^4}$ (4), $-\frac{I_z}{\sigma_0T_1^4}$ (5) as functions of position in the channel.

Calculations show that the quantity $h^r_{V-F}(z)$ does not vary over the middle half of the channel, but drops to 2.6% of its midchannel value at the endpoints.

The mathematical expression for the computation equation using a simplified scheme coincides with Eq. (14), but the problem of choice of coefficients σ_a in the theory of the simplified method has not been resolved. The last term in Eq. (14) defines the quantity of heat liberated by an elemental volume to the entire heating surface, and in the simplified method it represents the value of the local thermal loading of the heating surface. Use of the simplified method introduces errors in calculating local values of the thermal stress of the heating surface, indicating more nonuniformity than exists in reality (Fig. 3).

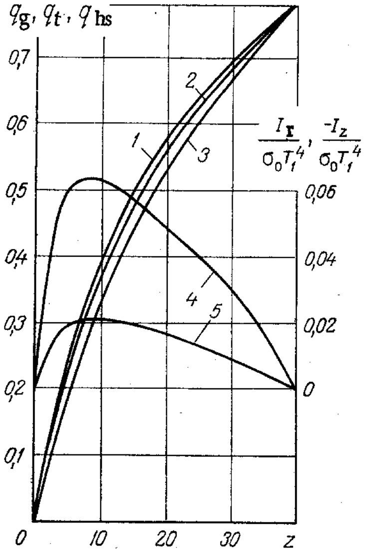

Fig. 2

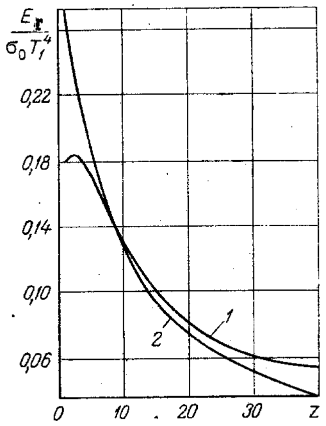

Fig. 3

Fig. 2. 1) Heat loss by gas; 2) heat liberation from gas to entire heating surface; 3) heating surface heat sensitivity; 4) $I_r/\sigma_0 T_1^4$; 5) $-I_z/\sigma_0 T_1^4$ vs position along channel. z, zone.

Fig. 3. Quantity $E_r/\sigma_0 T_1^4$ vs position along channel: 1) exact method; 2) simplified method.

In practice, Eq. (14) can be recommended for calculation of radiant heat exchange. The temperature of the medium along the length of the chamber can be found with sufficient accuracy with Eq. (14). Local values of heating surface thermal loading can be found with the zonal method from the specified temperature field, using normal methods for calculating radiant heat exchange. Thus, we avoid the necessity of solving a complex problem, the determination of radiant heat exchange in a moving medium with unknown temperature field.

Returning to Fig. 2, we note that with use of the method described above, curves 1 and 2 coincide. Using the older simplified method, curves 1, 2, and 3 coincide.

NOTATION

W, water equivalent of gas flow; f, flow section; T, gas temperature; T_{ex}, temperature of exiting gases; T_s, temperature of heating surface; T_1, gas temperature at channel inlet; $\theta_{ex} = T_{ex}/T_1$, $\theta_s = T_s/T_1$, dimensionless temperatures; q_c, chemical heat liberation per unit volume; $Q_c(z)$, chemical heat liberation in channel segment from 0 to z; ε_s, emissivity of heating surface; ε_a, apparent emissivity of chamber; σ_a, apparent radiant heat exchange coefficient; Π, channel perimeter section; ψ, ratio of heating surface to lateral surface of channel; E_r, resultant heat exchange density; $Q_{hs}(z)$, value of resultant heat exchange over channel segment from 0 to z; I_r, difference between oppositely directed radiant heat fluxes; $H^r_{V-F}(H, V)$, resolving coefficient for mutual radiant heat exchange between volume and surface.

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